OCR
Oxford Cambridge and RSA

Wednesday 18 May 2016 – Morning

AS GCE MATHEMATICS

4721/01 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4721/01
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

No calculator can be used for this paper

Answer **all** the questions.

1 **(i)** Simplify $(2x-3)^2 - 2(3-x)^2$. **[2]**

(ii) Find the coefficient of x^3 in the expansion of $(3x^2-3x+4)(5-2x-x^3)$. **[2]**

2 Express $\dfrac{3+\sqrt{20}}{3+\sqrt{5}}$ in the form $a+b\sqrt{5}$. **[4]**

3 Solve the simultaneous equations

$$x^2+y^2=34, \qquad 3x-y+4=0.$$ **[5]**

4 Solve the equation $2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0$. **[5]**

5 Express the following in the form 2^p.

(i) $(2^5 \div 2^7)^3$ **[2]**

(ii) $5 \times 4^{\frac{2}{3}} + 3 \times 16^{\frac{1}{3}}$ **[3]**

6 **(i)** Express $4+12x-2x^2$ in the form $a(x+b)^2+c$. **[4]**

(ii) State the coordinates of the maximum point of the curve $y=4+12x-2x^2$. **[2]**

7 **(i)** Sketch the curve $y=x^2(3-x)$ stating the coordinates of points of intersection with the axes. **[3]**

(ii) The curve $y=x^2(3-x)$ is translated by 2 units in the positive direction parallel to the x-axis. State the equation of the curve after it has been translated. **[2]**

(iii) Describe fully a transformation that transforms the curve $y=x^2(3-x)$ to $y=\frac{1}{2}x^2(3-x)$. **[2]**

8 A curve has equation $y=2x^2$. The points A and B lie on the curve and have x-coordinates 5 and $5+h$ respectively, where $h>0$.

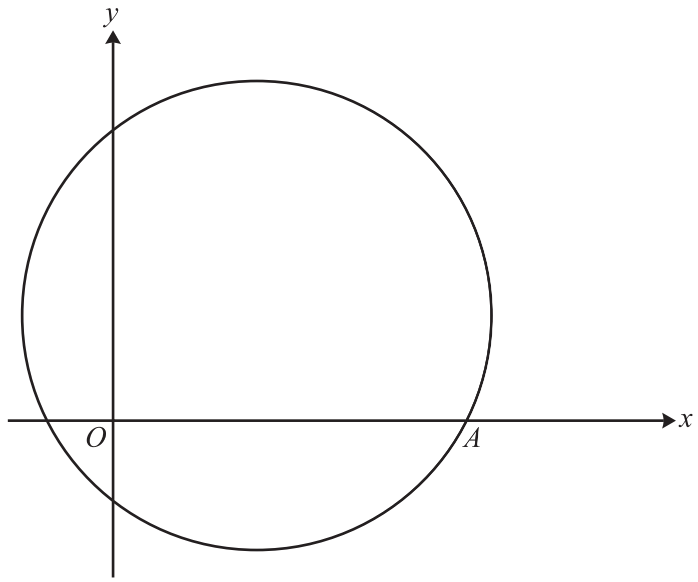
(i) Show that the gradient of the line AB is $20+2h$. **[3]**

(ii) Explain how the answer to part **(i)** relates to the gradient of the curve at A. **[1]**

(iii) The normal to the curve at A meets the y-axis at the point C. Find the y-coordinate of C. **[3]**

9 Find the set of values of k for which the equation $x^2+2x+11=k(2x-1)$ has two distinct real roots. **[7]**

10



The diagram shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

(i) Find the centre and radius of the circle. **[3]**

The circle crosses the positive x-axis at the point A.

(ii) Find the equation of the tangent to the circle at A. **[6]**

(iii) A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent. **[3]**

(iv) Another circle has centre at the origin O and radius r. This circle lies wholly inside the first circle. Find the set of possible values of r. **[2]**

11 The curve $y = 4x^2 + \frac{a}{x} + 5$ has a stationary point. Find the value of the positive constant a given that the y-coordinate of the stationary point is 32. **[8]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$4x^2 - 12x + 9 - 2(9 - 6x + x^2)$ $2x^2 - 9$	**M1** **A1** **[2]**	Square to get at least one 3/4 term quadratic Fully correct **www**	**ISW** after correct answer
1	(ii)		$-6x^3 - 4x^3$ -10	**B1** **B1** **[2]**	$-6x^3$ **or** $-4x^3$ soi **www** in these terms Condone $-10x^3$	Ignore other terms If only embedded in full expansion then award **B1B0**
2			$\frac{3+\sqrt{20}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ $\frac{-1+3\sqrt{5}}{9-5}$ $-\frac{1}{4}+\frac{3}{4}\sqrt{5}$	**M1** **B1** **A1** **A1** **[4]**	Attempt to rationalise the denominator – must attempt to multiply $\sqrt{20} = 2\sqrt{5}$ soi Either numerator or denominator correct and simplified to no more than two terms Fully correct and fully simplified. Allow $\frac{-1+3\sqrt{5}}{4}$, order reversed etc. **Do not ISW** if then multiplied by 4 etc.	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{5}+b$ **B1** $\sqrt{20} = 2\sqrt{5}$ soi **A1** Either a or b correct **A1** Both correct
3			$x^2 + (3x + 4)^2 = 34$ $10x^2 + 24x - 18 = 0$ $5x^2 + 12x - 9 = 0$ $(5x - 3)(x + 3) = 0$ $x = \frac{3}{5}, x = -3$ $y = \frac{29}{5}, y = -5$	**M1*** **A1** **M1dep*** **A1** **A1** **[5]**	Substitute for x/y or valid attempt to eliminate one of the variables Correct three term quadratic in solvable form Attempt to solve resulting three term quadratic Correct x values Correct y values	If x eliminated: $10y^2 - 8y + 290 = 0$ $5y^2 - 4y + 145 = 0$ $(5y - 29)(y + 5) = 0$ Award **A1 A0** for one pair correctly found from correct quadratic **Spotted solutions:** If **M0 DM0** **SC B1** $x = \frac{3}{5}, y = \frac{29}{5}$ **www** **SC B1** $x = -3, y = -5$ **www** Must show on both line and curve (Can then get 5/5 if both found **www** and exactly two solutions justified)

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td></td><td></td><td>Let $y^{\frac{1}{4}} = x$
$2x^2 - 7x + 3 = 0$
$(2x-1)(x-3) = 0$
$x = \frac{1}{2}, x = 3$
$y = \left(\frac{1}{2}\right)^4, y = 3^4$
$y = \frac{1}{16}, y = 81$

<u>Alternative by rearrangement and squaring:</u>
$2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0,\ 7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3$
$49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9,\ 37y^{\frac{1}{2}} = 4y + 9$
$16y^2 - 1297y + 81 = 0$
$(16y-1)(y-81) = 0$
$y = \frac{1}{16}, y = 81$

OR methods may be combined:
e.g. after $37y^{\frac{1}{2}} = 4y + 9$
$4y - 37y^{\frac{1}{2}} + 9 = 0$
$4x^2 - 37x + 9 = 0$
$(4x-1)(x-9) = 0$
$x = \frac{1}{4}, x = 9$
$y = \left(\frac{1}{4}\right)^2, y = 9^2$</td><td>M1*

M1dep*
A1
M1dep*
A1
[5]

M2*

A1
M1dep*
A1

M1*

M1dep*
A1
M1dep*
A1
[5]</td><td>Use a substitution to obtain a quadratic or factorise into two brackets each containing $y^{\frac{1}{4}}$
Correct method to solve resulting quadratic
Both values correct
Attempt to raise to the fourth power
Correct final answers

Rearrange and square both sides twice

Correct quadratic obtained
Correct method to solve resulting quadratic
Correct final answers

Rearrange, square both sides and substitute

Correct method to solve resulting quadratic

Attempt to square
Correct final answers</td><td>No marks if whole equation raised to fourth power etc.

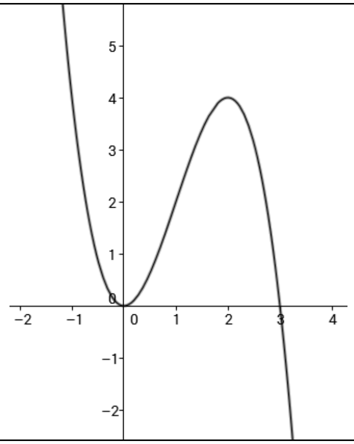
No marks if straight to formula with no evidence of substitution at start and no raising to fourth power/fourth rooting at end.

No marks if $y^{\frac{1}{4}} = x$ and then $2x - 7x^2 + 3 = 0$.

Spotted solutions:

If M0 DM0 or M1 DM0
SC B1 $y = 81$ www
SC B1 $y = \frac{1}{16}$ www
(Can then get 5/5 if both found www and exactly two solutions justified)</td></tr>
</table>

Question			Answer	Marks	Guidance	
5	(i)		$(2^{-2})^3$ or $2^{15} \div 2^{21}$	B1	Valid attempt to simplify	Correct use of either index law
			2^{-6}	B1 [2]	Correct answer. Accept $p = -6$.	$\left(\frac{1}{2}\right)^6$ **oe** is **B1**
5	(ii)		$5\times\left(2^2\right)^{\frac{2}{3}}+3\times\left(2^4\right)^{\frac{1}{3}}$	M1	Attempts to express both terms or a combined term as a power of 2	e.g. **Both** $4 = 2^2$ **and** $16 = 2^4$ **soi**
			$=5\times 2^{\frac{4}{3}}+3\times 2^{\frac{4}{3}}$ or $10\times 2^{\frac{1}{3}}+6\times 2^{\frac{1}{3}}$	B1	Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for **either** term	If **M0** **SC B1** for $8\times 16^{\frac{1}{3}}$ or $8\times 4^{\frac{2}{3}}$
			$=8\times 2^{\frac{4}{3}}$			
			$=2^{\frac{13}{3}}$	A1 [3]	Correct final answer	
6	(i)		$-2(x^2-6x-2)$	B1	or $a = -2$	$-2(x-3)^2-22$ **B1 B1 M0 A0**
			$=-2[(x-3)^2-2-9)]$	B1	$b = -3$	$-2(x-3)+22$ **4/4 (BOD)**
				M1	$4+2b^2$	$-2(x-3x)^2+22$ **B1 B0 M1 A0**
			$=-2(x-3)^2+22$	A1	$c = 22$	$-2(x^2-3)^2+22$ **B1 B0 M1 A0**
				[4]	If a, b and c found correctly, then **ISW** slips in format. **If signs of all terms changed at start, can only score SC B1** for fully correct working to obtain $2(x-3)^2-22$ If done correctly and then signs changed at end, do not **ISW**, award **B1B1M1A0**	$-2(x+3)^2+22$ **B1 B0 M1 A0** $-2x(x-3)^2+22$ **B0 B1 M1 A0** $-2(x^2-3)+22$ **B1 B0 M1 A0**
6	(ii)		(3, 22)	B1ft B1ft [2]	Allow follow through "– their b" Allow follow through "their c"	May restart. Follow through marks are for **their final answer** to (i)

Question		Answer	Marks	Guidance	
7	(i)		**B1** **B1** **B1** **[3]**	Negative cubic with a max and a min Cubic that meets y-axis at (0, 0) only Double root at (0,0) and single root at (3, 0) and no other roots	For **first mark** must clearly be a cubic – must not stop at or before x axis, do not allow straight line sections drawn with a ruler/tending to extra turning points etc. Must not be a finite plot.
7	(ii)	$y=(x-2)^2(5-x)$ or $y=3(x-2)^2-(x-2)^3$	**M1** **A1** **[2]**	Translates curve by +2 or – 2 parallel to the x-axis; must be consistent Fully correct, must have "$y=$". **ISW** expansions	e.g. for M1 $(x-2)^2(3-(x-2))$ but not $(x-2)^2(3-x-2)$
7	(iii)	Stretch Scale factor one-half parallel to the y-axis	**B1** **B1** **[2]**	**Must** use the word "stretch" **Must** have "factor" or "scale factor". For "parallel to the y axis" allow "vertically", "in the y direction".	Do **not** accept "in/on/across/up the y axis". Allow second B1 after "squash" etc. but not after "translate" etc.
8	(i)	$y_1=50$, $y_2=2(5+h)^2$ $\dfrac{(50+20h+2h^2)-50}{(5+h)-5}$ $20+2h$	**B1** **M1** **A1** **[3]**	Finds y coordinates at 5 and $5+h$ Correct method to find gradient of a line segment; at least 3/4 values correct Fully correct working to give answer **AG**	Need not be simplified
8	(ii)	e.g. "As h tends to zero, the gradient will be 20"	**B1** **[1]**	Indicates understanding of limit **See Appendix 2 for examples**	e.g. refer to h tending to zero or substitute $h=0$ into $20+2h$ to obtain gradient at A
8	(iii)	Gradient of normal $=-\dfrac{1}{20}$ $y-50=-\dfrac{1}{20}(x-5), x=0$ 50¼	**B1** **M1** **A1** **[3]**	 Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\dfrac{201}{4}, \dfrac{1005}{20}$	 Any correct method e.g. labelled diagram.

Question			Answer	Marks	Guidance	
9			$x^2 + (2 - 2k)x + 11 + k = 0$ $(2 - 2k)^2 - 4(11 + k)$ $4k^2 - 12k - 40 > 0$ $k^2 - 3k - 10 > 0$ $(k - 5)(k + 2)$ $k < -2, k > 5$	**M1*** **M1dep*** **A1** **M1dep*** **A1** **M1dep*** **A1** **[7]**	Attempt to rearrange to a three-term quadratic Uses $b^2 - 4ac$, involving k and not involving x Correct simplified inequality obtained **www** Correct method to find roots of 3-term quadratic 5 and – 2 seen as roots $b^2 - 4ac > 0$ and chooses "outside region" Fully correct, strict inequalities.	Each Ms depend on the previous M $-2 > k > 5$ scores **M1A0** Allow "$k < -2$ or $k > 5$" for A1 **Do not allow** "$k < -2$ **and** $k > 5$"
10	(i)		Centre of circle (4, 3) $(x-4)^2 - 16 + (y-3)^2 - 9 - 20 = 0$ $r^2 = 45$ $r = \sqrt{45}$	**B1** **M1** **A1** **[3]**	Correct centre $(x \pm 4)^2 - 4^2$ and $(y \pm 3)^2 - 3^2$ seen (or implied by correct answer) $\sqrt{45}$ or better **www**	Or $r^2 = 4^2 + 3^2 + 20$ **soi** **ISW** after $\sqrt{45}$
10	(ii)		At A, $y = 0$ so $x^2 - 8x - 20 = 0$ $(x - 10)(x + 2) = 0$ A = (10, 0) Gradient of radius = $\dfrac{3-0}{4-10} = -\dfrac{1}{2}$ Gradient of tangent = 2 $y - 0 = 2(x - 10)$ $y = 2x - 20$	**M1** **A1** **M1** **B1** **M1** **A1** **[6]**	Valid method to find A e.g. put $y = 0$ and attempt to solve quadratic (allow slips) or Pythagoras' theorem Correct answer found Attempts to find gradient of radius (3 out of 4 terms correct for their centre, their A) Equation of line **through their A**, any non-zero gradient Correct answer in any three-term form	Alterative for finding gradient: **M1** Attempt at implicit differentiation as evidenced by $2y\dfrac{dy}{dx}$ term **A1** $2x + 2y\dfrac{dy}{dx} - 8 - 6\dfrac{dy}{dx} = 0$ **and** substitution of (10, 0) to obtain 2.
10	(iii)		A' = (–2, 6) $y - 6 = 2(x + 2)$ $y = 2x + 10$	**B1** **M1** **A1** **[3]**	Finds the opposite end of the diameter Line through their A' parallel to their line in (ii) Correct answer in any three-term form	 Not through centre of circle
10	(iv)		OC = $\sqrt{3^2 + 4^2} = 5$ $(0 <)\ r < \sqrt{45} - 5$	**M1** **A1** **[2]**	Attempts to find the distance from O to their centre **and subtract from their radius** Correct inequality, condone $\le$	**ISW** incorrect simplification

Question			Answer	Marks	Guidance	
11			$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} **soi**	
			$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
				A1	Fully correct	
			At stationary point, $8x - ax^{-2} = 0$	M1	Sets their derivative to 0	
			$a = 8x^3$ **oe**	A1	Obtains expression for a in terms of x, or x in terms of a **www**	$x = \frac{\sqrt[3]{a}}{2}$ **oe**, $a = 18x$ **oe** also fine
			When $a = 8x^3$, $y = 32$ $32 = 4x^2 + 8x^2 + 5$	M1	Substitutes their expression and 32 into equation of the curve to form single variable equation	
			$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	or expression for a e.g. $a^{\frac{2}{3}} = 9$
			$a = 27$	A1 **[8]**	Obtains correct value for a. Ignore –27 given as well.	
			OR			
			$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} **soi**	
			$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
				A1	Fully correct	
			$32 = 4x^2 + ax^{-1} + 5$	M1	Substitutes 32 into equation of the curve to find expression for a	
			$a = 27x - 4x^3$	A1	Obtains expression for a in terms of x **www**	
			At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$	M1	Sets derivate to zero **and** forms single variable equation	
			$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	
			$a = 27$	A1	Obtains correct value for a. Ignore –27 given as well.	

APPENDIX 1

Allocation of method mark for solving a quadratic

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ earns **M1** (6 for c instead of −6)

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ **M0** (2 sign errors: initial sign and c incorrect)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times -6}$$ **M0** ($2c$ on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last (complete) attempt.

APPENDIX 2 – this section contains additional subject specific information

Example responses to 8ii	
h is zero so the gradient is 20 **B1**	The gradient at A is 20 **B0**
At A x = 5, *h* = 0 so gradient equals 20 **B1**	The gradient at A is 20 so *h* = 0 **B0**
As *h* approaches 0, the gradient of AB approaches 20 which is the gradient of A **B1**	At A, gradient is 20 so it's 2*h* more **B0**
As *h* were infinitely small, 20 + 2*h* is the same as the gradient at A, otherwise it's greater than the gradient at A **B1**	$\frac{dy}{dx} = 20$, so it is the gradient of A plus a bit more **B0**
The smaller *h* is the closer the gradient of AB is to the gradient of the curve at A **B1**	2*h* + 20 = 20 so *h* = 0 **B0**
As *h* tends to zero the gradient gets closer and closer to the actual value **B1**	They're getting closer to each other **B0**
The gradient of AB tends to the gradient of the tangent of the curve as *h* tends to zero **B1**	
The answer of (i) is converging towards the gradient at A **B1**	